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328352(28)

B. E. (Third Semester) Examination, April-May 2021

(New Scheme)

(Et & T Branch)

PROBABILITY and RANDOM VARIABLES

Time Allowed : Three hours

Maximum Marks : 80

Minimum Pass Marks : 28

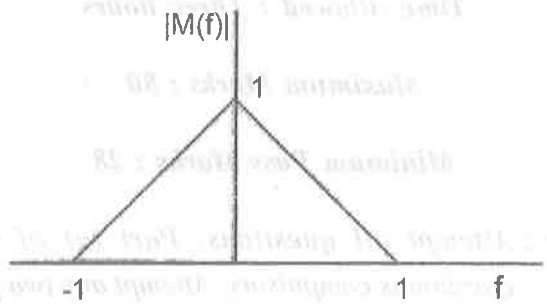
Note : *Attempt all questions. Part (a) of each question is compulsory. Attempt any two parts from (b), (c) and (d). All questions carry equal marks.*

Unit-I

1. (a) State and prove the convolution property of Fourier Transform. 2

(b) A train of rectangular pulses, making excursions from zero to 1 volt, have a duration of 2 μ sec and are separated by intervals of 10 μ sec. Assume that the center of one pulse is located at $t = 0$, write the exponential Fourier series for the pulse train and plot the spectral amplitude as a function of frequency. Also draw the envelope. 7

(c) A waveform $m(t)$ has a Fourier transform $M(f)$ whose magnitude is as shown.



(i) Find the normalized energy content of the waveform.

(ii) Calculate the frequency f_1 such that one-half of the normalized energy is in the frequency range $-f_1$ to f_1. 7

(d) Define Cross-correlation. Find the cross-correlation of $v_1(t) = \sin w_0 t$ and $v_2(t) = \cos w_0 t$. 7

Unit-II

2. (a) State and prove the Baye's theorem. 2

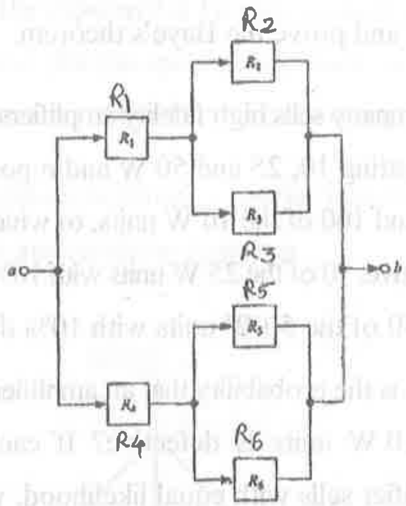
(b) A company sells high fidelity amplifiers capable of generating 10, 25 and 50 W audio power. It has on hand 100 of the 10 W units, of which 15% are defective, 70 of the 25 W units with 10% defective, and 30 of the 50 W units with 10% defective. What is the probability that an amplifier sold from the 10 W units is defective? If each wattage amplifier sells with equal likelihood, what is the probability of a randomly selected unit being 50 W and defective? What is the probability that a unit randomly selected for sale is defective? 7

(c) In a communication system the signal send from point a to point b arrives by two paths as shown in figure. All repeaters fail independently of each other. The probability of failing of repeaters are

$P(R_1) = 0{\cdot}005, P(R_2) = P(R_3) = P(R_4) = 0.01,$

and $P(R_5) = P(R_6) = 0{\cdot}05$.

Find the probability that the signal will not arrive at point b. 7



(d) Spacecrafts are expected to land in a prescribed recovery zone 80% of the time. Over a period of time, six spacecrafts land.

(i) Find the probability that none of them lands in the prescribed zone.

(ii) Find the probability that at least one lands in the prescribed zone.

(iii) The landing program is called successful if the probability is 0·9 or more that 3 or more out of the six spacecrafts will land in the prescribed zone. Is the program successful? 7

Unit-III

3. (a) Define Probability Density function. State the various properties of Probability density function. 2

(b) Consider an experiment of "Rolling of 2 dice". Take the random variable as "the sum of two numbers that show on the dice" and find the corresponding cumulative distribution function. 7

(c) Certain random variable has CDF as given

$$F_X(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ Kx^2 & \text{for} \quad 0 \le x \le 10 \\ 100K & \text{for} \quad x > 10 \end{cases}$$

Find the value of K and the corresponding pdf. Also find $P\{X \le 5\}$ and $P\{5 \le X \le 7\}$ and draw the plots of CDF and pdf. 7

(d) The lifetime of a system expressed in weeks is a Rayleigh random variable X for which

$$f_X(x) = \begin{cases} \left(\frac{x}{200}\right)\exp\left(-\frac{x^2}{400}\right) & \text{for } x \ge 0 \\ 0 & \text{for } x < 0 \end{cases}$$

(i) What is the probability that the system will not last a full week?

(ii) What is the probability that the system lifetime will exceed one year? 7

Unit-IV

4. (a) Define Random process. 2

(b) Consider the random process

$V(t) = \cos(w_0 t + \phi)$ where ϕ is a random variable which is uniformly distributed in the range of $(-\pi, \pi)$.

(i) Show that the 1st and 2nd moments of $V(t)$ are independent of time.

(ii) Show that $V(t)$ is wide-sense stationary process. 7

(c) Let two random processes $X(t)$ and $Y(t)$ be defined as :

$$X(t) = A\cos(w_0 t) + B\sin(w_0 t)$$

$$Y(t) = B\cos(w_0 t) - A\sin(w_0 t)$$

Where A and B are random variables and w_0 is a constant. $X(t)$ is Wide-Sense Stationary since A and B are uncorrelated, zero-mean Random variables with the same variances. With the same constraints on A and B, $Y(t)$ is also Wide-Sense Stationary. Show that $X(t)$ and $Y(t)$ are jointly Wide-Sense Stationary. 7

(d) Explain the Poisson Random Process. 7

Unit-V

5. (a) Define Cross-Power density spectrum. 2

(b) A wide-sense stationary process $X(t)$ has an auto-correlation function

$$R_{XX}(\tau) = \left\{A_0\left[1-\left(|\tau|/T\right)\right] \text{ for } -T <= T \text{ and } 0 \text{ elsewhere}\right\}$$

Where $T > 0$ and A_0 is a constant. Determine the power spectrum. 7

(c) Determine the cross-correlation function corresponding to the cross-power density spectrum

$$\mathscr{S}_{XY}(\omega) = \begin{cases} a + \dfrac{jb\omega}{W} & -W < \omega < W \\ 0 & \text{elsewhere} \end{cases}$$

where $\alpha > 0$ is a constant. 7

(d) What is the relationship between cross-power spectrum and cross-correlation function of $X(t)$ and $Y(t)$. 7